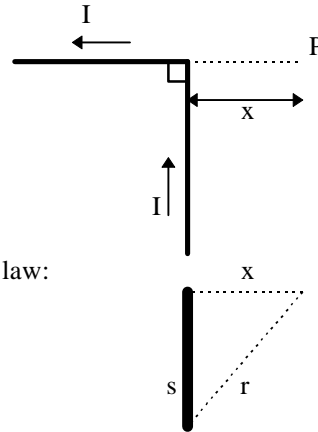**Problem 19.29**: Determine the magnetic field at a point, *P*,
that is a distance of *x* away from the corner of an infinitely
long wire that is bent at a right angle, as shown in Figure
19.37. The wire carries a steady current, *I*.

For the vertical piece of wire, we use the Biot-Savart law:

$$d\vec{B} = \frac{m_o I}{4p} \frac{d\vec{s} \times \hat{r}}{r^2}$$

If we adopt a standard coordinate system (+*x* to the right, +*y* upward, +*z* out of the page), we can make
some conversions based on the second diagram:

$$r = \sqrt{s^2 + x^2} \; ; \; \hat{r} = \frac{1}{\sqrt{s^2 + x^2}}(x\hat{i} + s\hat{j}) \; ; \; d\vec{s} = ds\hat{j}$$

Inserting these into the Biot-Savart law gives:

$$d\vec{B} = \frac{m_0 I}{4p} \frac{(ds\hat{j}) \times (x\hat{i} + y\hat{j})}{(s^2 + x^2)^{\frac{3}{2}}}$$

The cross-product can now be performed quite easily. We can now integrate both sides:

$$\vec{B} = -\frac{m_0 Ix}{4p} \hat{k} \int_{-\infty}^{0} \frac{ds}{(s^2 + x^2)^{\frac{3}{2}}}$$

This integral is listed in Table B.5, page A.26, in the back of Serway, so we get

$$\vec{B} = -\frac{m_0 Ix}{4p} \hat{k} [\frac{s}{x^2\sqrt{s^2 + x^2}}]_{-\infty}^{0}$$

Evaluating this expression at the limits gives

$$\vec{B} = -\frac{m_0 I}{4px} \hat{k}$$

For the horizontal piece of wire, $d\vec{s} \times \hat{r} = 0$, so there is no contribution. Thus, the total magnetic field
at *P* is equal to the field due to the vertical section:

$$\vec{B} = -\frac{m_0 I}{4px} \hat{k}; \quad B = \frac{m_0 I}{4px} \text{ , into the page.}$$

Problem 19.32: Consider the current-carrying loop shown in Figure 19.40, formed of radial lines and segments of circles whose centers are at point P. Find the magnitude and direction of the magnetic field B at P.

The radial lines will not contribute, since $d\vec{s} \times \hat{r} = 0$. A circle segment of radius r gives an infinitesimal magnetic field as given by the Biot-Savart law:

$$d\vec{B} = \frac{m_0 I}{4p} \frac{d\vec{s} \times \hat{r}}{r^2}$$

We can define the arc length by $s = rq$, $ds = rdq$. We can insert this into the Biot-Savart law and then integrate to find the arc's contribution to the field magnitude:

$$B = \frac{m_0 I}{4pr} q = \frac{m_0 I}{12r}, \quad \text{since} \quad q = \frac{2p}{6}$$

When we actually plug in for the two arcs, we must be careful to notice that the field contributions are in opposite directions. We can find the directions through use of the right-hand rule. The total magnetic field at P is

$$B = \frac{m_0 I}{12}(\frac{1}{a} - \frac{1}{b}) \text{ , out of the page.}$$

Problem 19.35: At what distance from a long, straight wire carrying a current of 5A is the magnetic field due to the wire equal to the strength of the Earth's field -- approximately $5 \times 10^{-5} T$?

Here, we can go back to Example 19.4, which gives an expression for the magnetic field near a long, straight wire:

$$B = \frac{m_0 I}{2pa}$$

In this case, however, we are given the magnetic field and asked to find the distance from the wire. This requires only a simple algebraic manipulation:

$$a = \frac{m_0 I}{2pB}$$

Inserting numbers, we find that a = 2 cm.

Problem 19.36: For the arrangement shown in Figure 19.42, the current in the long, straight conductor has the value $I_1 = 5A$ and lies in the plane of the rectangular loop, which carries a current of $I_2 = 10A$. The dimensions are c = 0.1 m, a = 0.15 m, and l = 0.45 m. Find the magnitude and direction of the net force exerted on the rectangle by the magnetic field of the straight current-carrying conductor.

The magnetic forces on the top and bottom sections of the loop cancel each other (convince yourself that they are not zero). The other two sections contribute forces which are given by equation [19.20]:

$$\frac{F}{l} = \frac{m_0 I_1 I_2}{2pr}$$

We can solve this equation for the force on the left and right sections. The directions of these forces can be determined by using the right-hand rule. This yields a net force given by:

$$\vec{F} = \frac{m_0 I_1 I_2 l}{2p}(\frac{1}{c} - \frac{1}{c+a}) \text{ , to the left.}$$

Inserting the numbers, we find that:

$$\vec{F} = 2.7 \times 10^{-5} N \text{ , to the left.}$$